# Probabilistic World Modeling with Asymmetric Distance Measure

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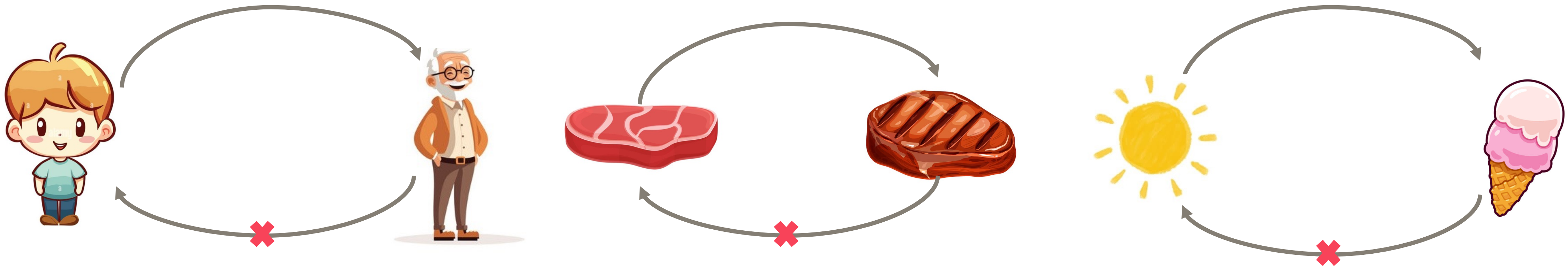
# What is a good representation for planning?

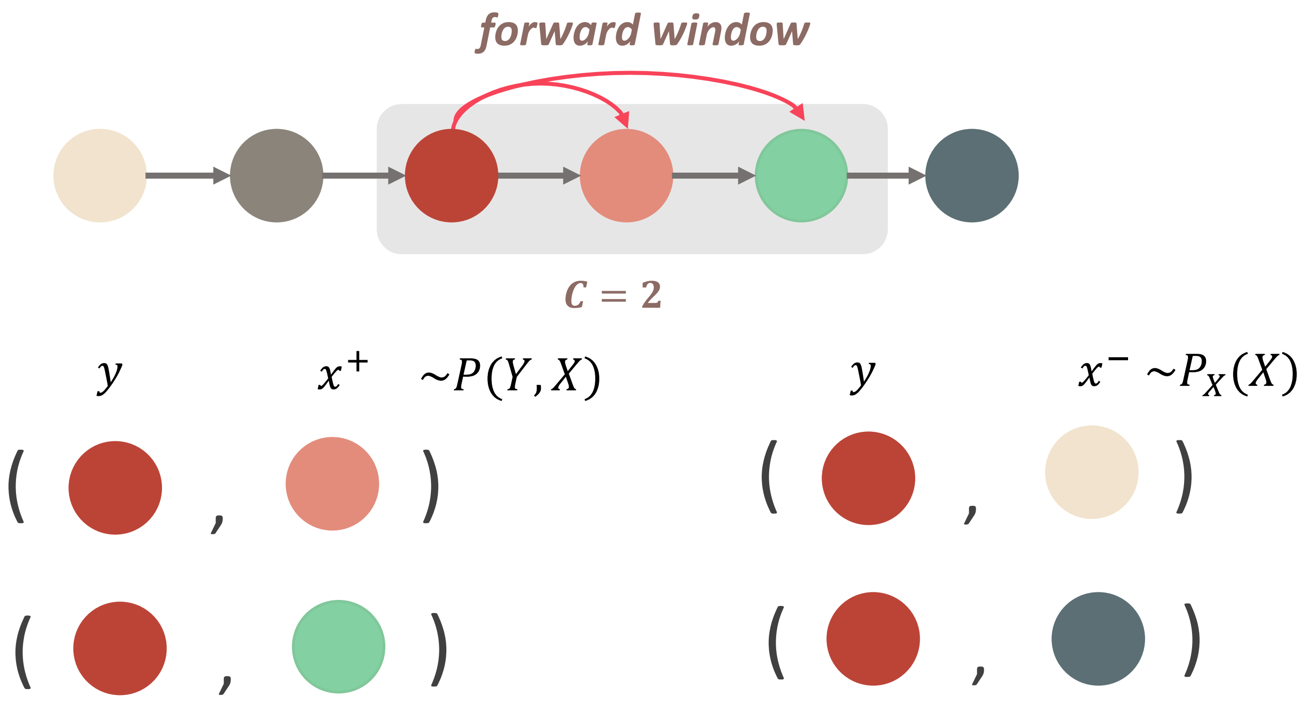
- An ***asymmetric*** distance function reflecting the state transitions



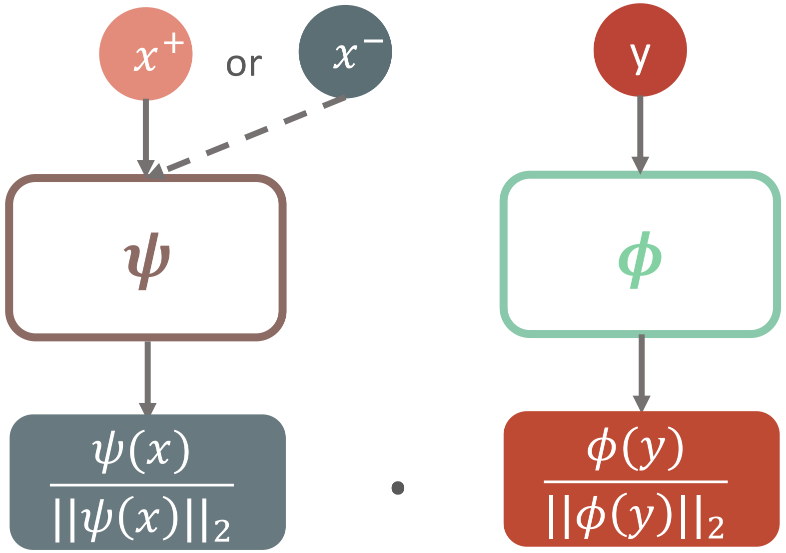
**Irreversible** transitions

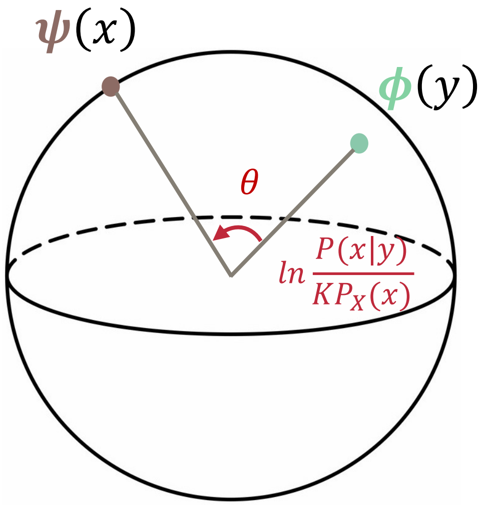
# Asymmetric contrastive learning

- Applying contrastive learning to the Markov chain of world dynamics



# Asymmetric contrastive learning



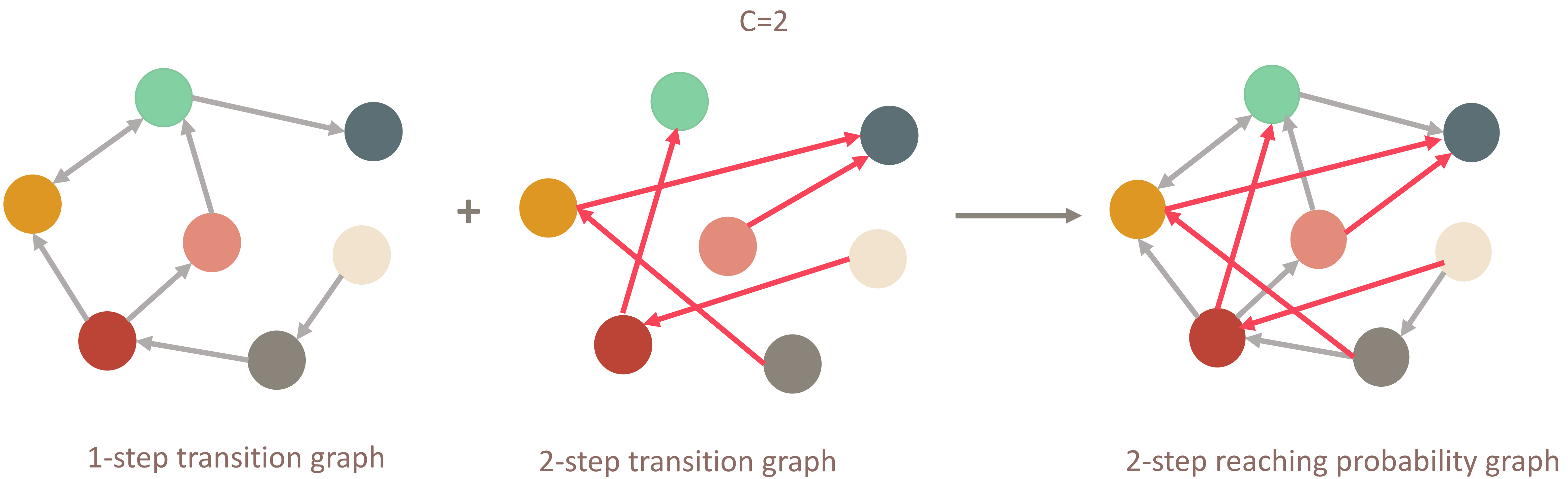




embedding space

$$\exp\left(cos\big(\psi(x), \phi(y)\big)\right) = \frac{P(x|y)}{KP_X(x)}$$ **Reaching probability from y to x**

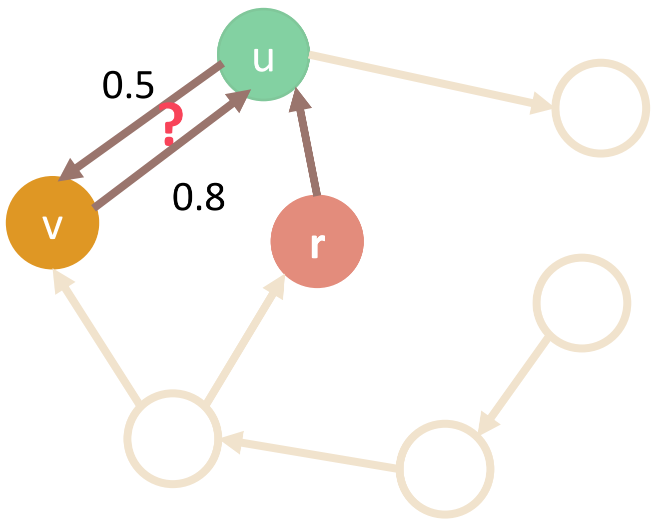
**Two separate encoders:** $cos\big(\psi(x), \phi(y)\big) \neq cos\big(\psi(y), \phi(x)\big)$

# C-step geometric abstraction

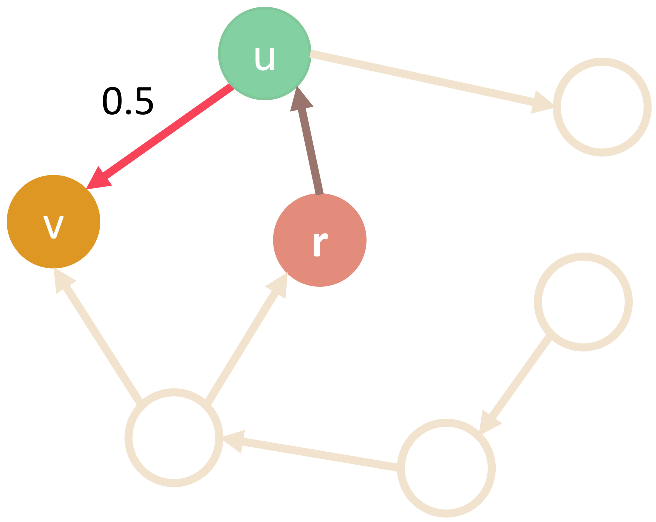



$$P(X = x|Y = y) \approx \frac{1}{C}\sum_{t=1}^{C}(P^t)_{yx}$$

# Reference state conditioned distance measure




Extract pairwise distances $d_r(u, v)$

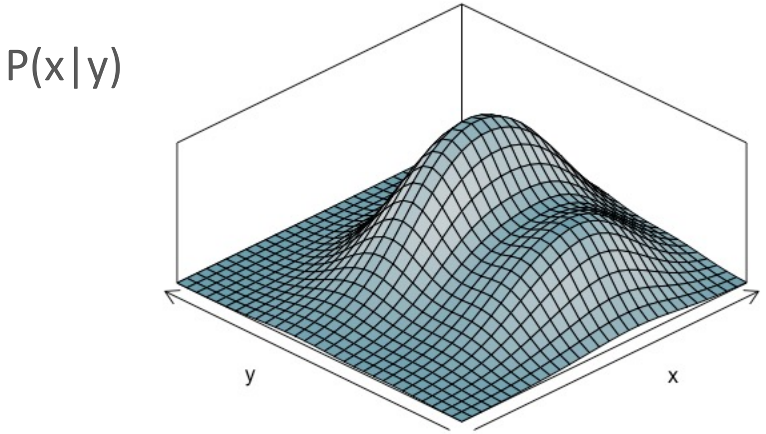


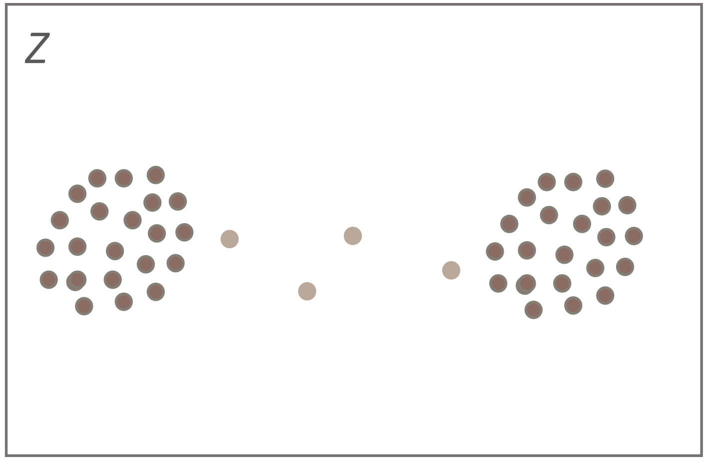

$$\phi(r) \to \psi(u) \quad \leq \quad \phi(r) \to \psi(v)$$

Choose $\phi(u) \to \psi(v)$

# Subgoal discovery

- Define ***subgoals*** as the states that **reduce** pairwise reaching probability, as perceived from the agent's current state.
- Identified using DBSCAN on the latent point density estimated according to $d_r(u, v)$.

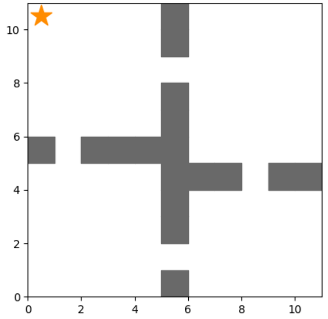


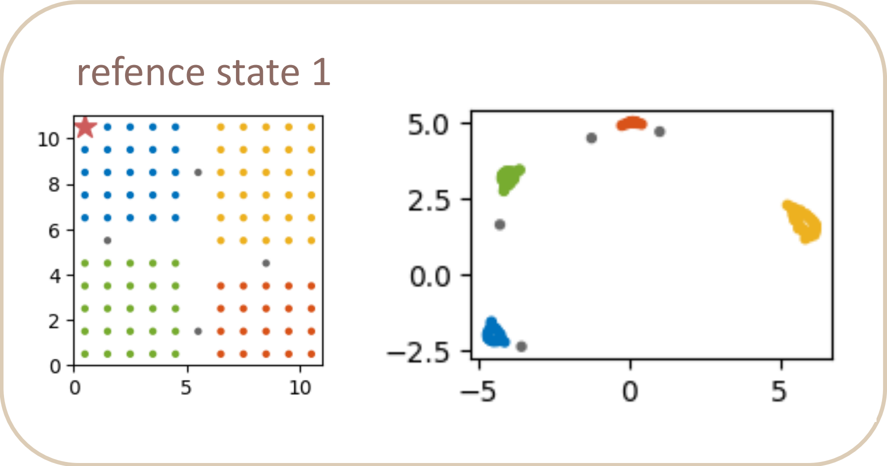



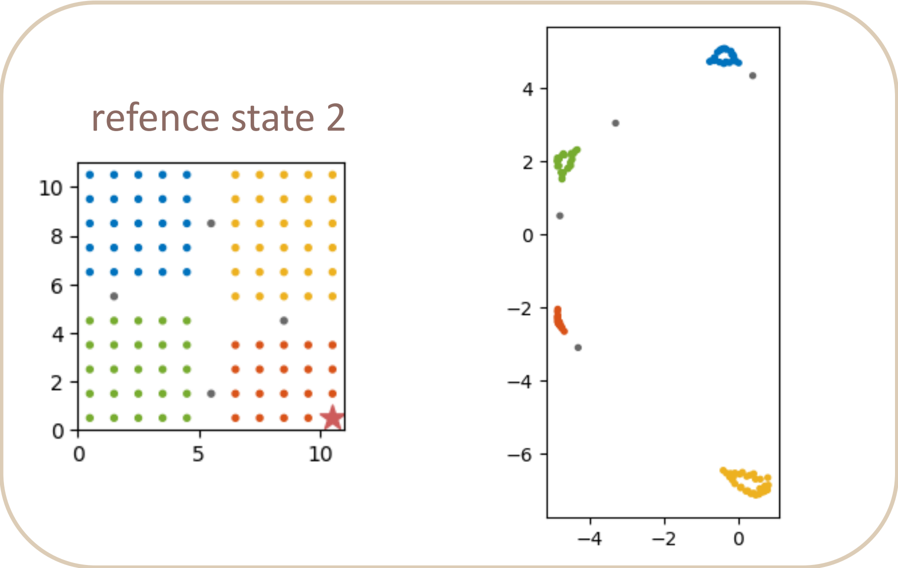

**probability density** in the original state space

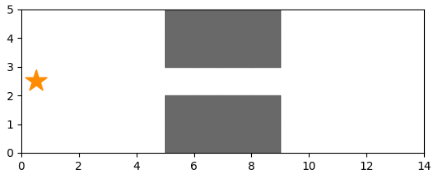
**point density** in the latent space

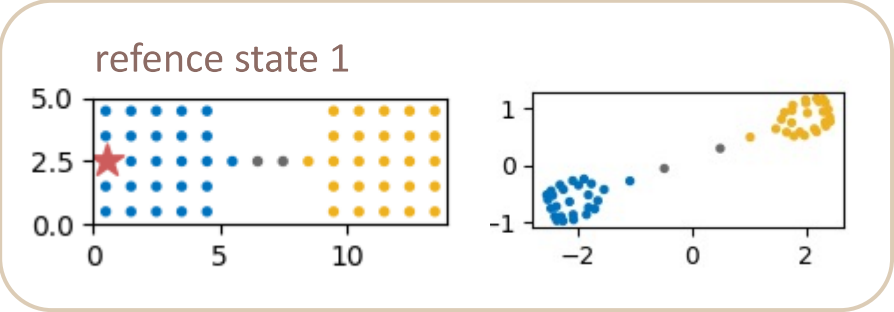
# Experiments

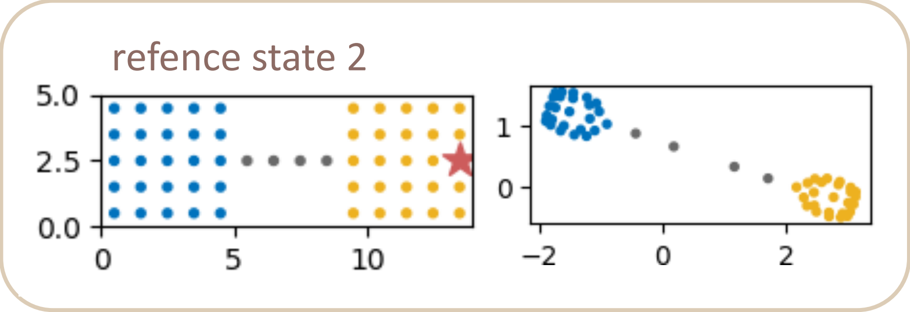




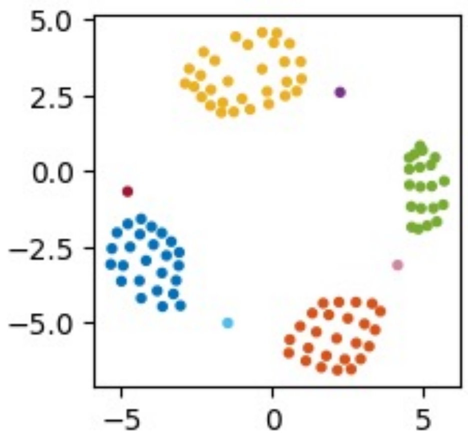


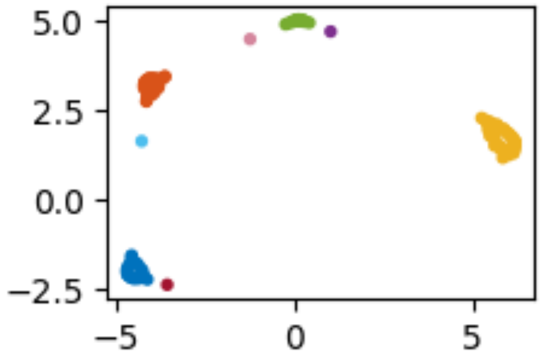

Subgoals identified as gray states

# The effect of step size C

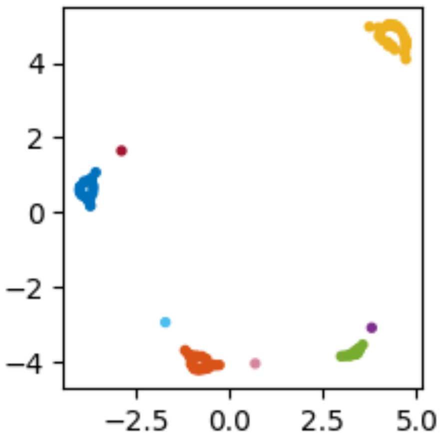
The rooms and doorways are separated further apart in the representation space as C goes up.




*C=1*




*C=16*




*C=64*

# Summary

- Contrastive learning with two separate encoders gives rise of an **asymmetric similarity function** that encodes state reaching probability.
- Contrastive learning embeds a **geometric abstraction** of the original Markov transition graph.
- These two geometric properties together enable us to find perspective-conditioned subgoals easily.

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